B. Math Analysis 1 Mid Semester 15-09-2005 . Answer all the questions (8 x 5 = 40).

If you are using a theorem/result proved in the class state it correctly. All answers need justification.

1) Show that between any two real numbers there are infinitely many rational numbers. State all the properties of real numbers that you will be using in establishing this.

2) Show that the set $A = \{a + ib : a, b \in [0, 1]\}$ is not a countable set.

3) Compute $limsup$ and $liminf$ of the sequence $\{(x_n)^{\frac{1}{n}}\}_{n\geq 1}$, where x_n is defined by $x_n = \frac{1}{2^{\frac{n+1}{2}}}$ when n is odd and $x_n = \frac{1}{3^{\frac{n}{2}}}$ when n is even.

4) Let $\{x_n\}_{n\geq 1}$ be a bounded sequence, suppose every convergent subsequence of $\{x_n\}_{n\geq 1}$ converges to a real number l. Show that $lim x_n = l$.

5) Compute the following limits. Give complete arguments.

a) $lim_{n\to\infty}(p)^{\frac{1}{n}}$

b) $lim_{n\to\infty} \frac{1}{n}(1 + \frac{1}{2} + ... + \frac{1}{2^{n-1}})$

6) State and prove Leibnitz' alternating series test for convergence of a series.

7) Let $\sum a_n$ be a convergent series of positive terms. Show that $\sum a_n^2$ is a convergent series. What can you say about the series $\sum a_n^{\frac{1}{2}}$?

8) Test the following series for absolute convergence or divergence

a) $x_n = (\frac{sin n}{n})^n$ (here we assume that the sine function is defined as in your +2 course).

b) The series $\sum x_n$ is defined by $x_n = \frac{1}{2^{\frac{n+1}{2}}}$ when n is odd and $x_n = \frac{1}{3^{\frac{n}{2}}}$ when n is even.